Uncertain Nonlinearities in Groundwater Flow Models

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The equations for groundwater fluid flow have been established for many years[1]. Despite their wide applicability and the great societal importance, these equations include large factors of uncertainty. One of the least constrained parameters in underground fluid flow is the permeability since its approximate value[2] and functional form[3,4] is largely uncertain. The dependence of permeability on porosity is non-trivial and it is usually expressed via the Carman-Kozeny porosity-permeability relationships.

There are substantial uncertainties concerning these nonlinear dependencies; for instance, for the function describing the permeability in terms of the porosity, a power-law relationship is commonly used[5]. With the practically available information, however, the corresponding exponent cannot be pinpointed exactly, but can only be restricted to a certain interval. Its value has a strong influence on the formation of nonlinear waves and of porosity channels in the resulting groundwater flow (Fig.1).

For uncertainty quantification in partial differential equations, sophisticated numerical methods are available[5]. Groundwater flows are among the most established applications motivating these developments, but generally they are considered in a linearized stationary setting, where such decisive nonlinear and intrinsically time-dependent effects are neglected. The aim of this project is thus to develop efficient numerical methods for quantifying the effects of uncertain nonlinearities in equations describing groundwater flows.

To this end, the doctoral candidate is expected to work on the mathematical analysis of such more complicated groundwater models for different parameterizations of the nonlinear terms, to devise new methods for uncertainty propagation that exploit the particular structure of the equations, and to devise Bayesian inverse methods for identifying parameter values for the nonlinearities that are compatible with empirical data. The project is particularly suitable for candidates who have a strong interest both in numerical analysis and in mathematical modelling.

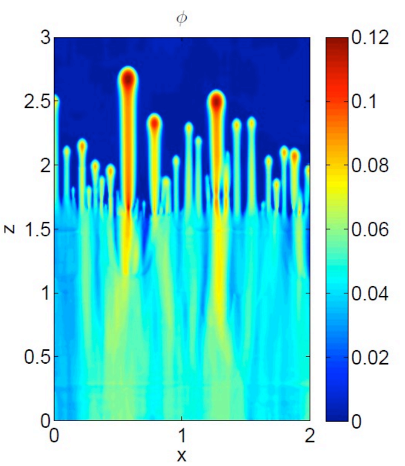

Figure 1. Channeling instability in non-linear porous flow. The color scale corresponds to the amount of fluid in the porous medium.

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